807 AP Physics Schmidt

Name: ______________________ Block: _______

U6H7 Chapter 12: Read section 12-9 Rolling without slipping

$v = R\omega$; $a = R\alpha$; $K.E_{Total} = \frac{1}{2}Mv_C^2 + \frac{1}{2}I\omega^2; K.E + P.E = Constant$

1. A uniform hoop, disk and sphere having the mass M and the same radius R, are released simultaneously from rest at the top of a ramp whose length L is 2.5 m and whose ramp angle is 12°.
(a) Which object reaches the bottom first? (sphere)
(b) How fast are the objects moving at the ramp's bottom? (2.7m/s; 2.6m/s; 2.3m/s)

2. A body of radius R and mass m is rolling smoothly with speed v on a horizontal surface. It then rolls up a hill to a maximum height h. (a) If $h = 3v^2/4g$, what is the body's rotational inertia about the rotational axis through its center of mass? (b) What might the body be? (Disc)

3. A solid cylinder of radius 10cm and mass 12 kg starts from rest and rolls without slipping a distance of 6.0cm down a house roof that is inclined at 30°. (a) What is the angular speed of the cylinder about its center as it leaves the house roof? (b) The outside wall of the house is 5.0 m high. How far from the edge of the roof does the cylinder hit the level ground? (6.32 rad/s, 0.53m)

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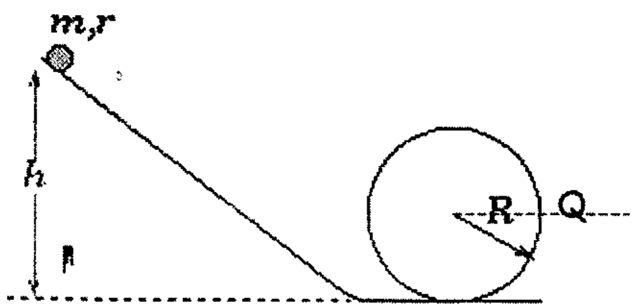
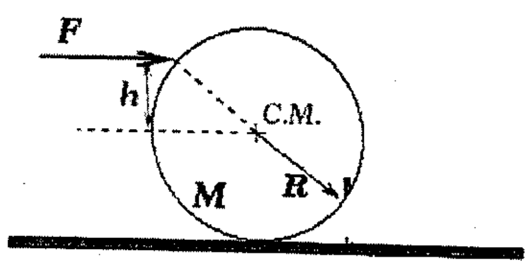
4. A small solid marble of mass m and radius r will roll without slipping along the loop-the-loop track if it is released from rest somewhere on the straight section of the track. (a) From what minimum height h above the bottom of the track must the marble be released to ensure that it does not leave the track at the top of the loop? (The radius of the loop is R; assume R>>r) (b) If the marble is released from height 6R above the bottom of the track, what is the horizontal component of the force acting on it at point Q? (h= 2.7 R, 50mg/7)



5. A cue ball of mass M and radius R is hit such that the force is directed horizontally along a line which is a distance h above the center line of the ball (see the figure). The ball is on a horizontal, level surface. What is the value of h such that the ball immediately begins rolling without slipping after the force is applied by a cue stick? Ignore Friction. (HINT: You will need to set up equations pertaining to linear and rotational motion and solve for h) (h = 2R/5)



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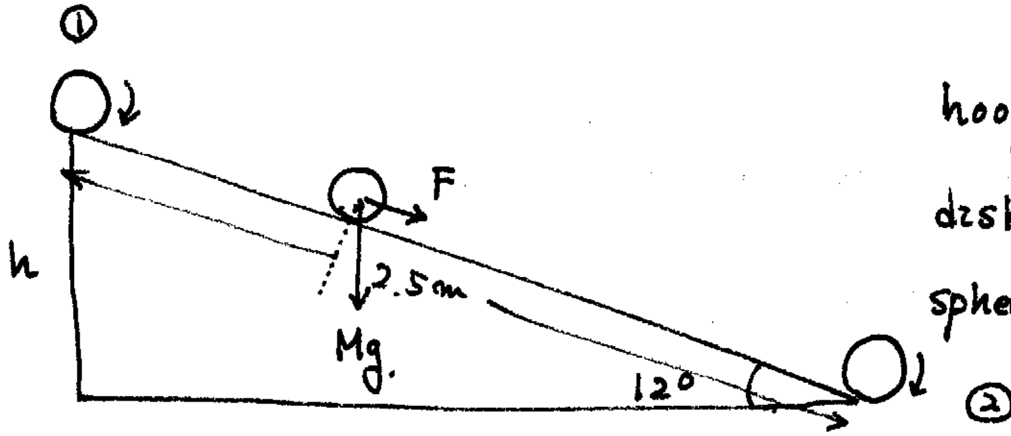
1.



hoop $I_h = MR^2$

disk $I_D = \frac{1}{2}MR^2$

sphere $I_S = \frac{2}{5}MR^2$

$\therefore h = 2.5 \cdot \sin 12° = 0.52\ m.$

(a) $a = R \cdot \alpha. \longrightarrow \alpha = \frac{a}{R}$

$$\tau = I \cdot \alpha = I \cdot \frac{a}{R}.$$

$$\therefore a = \frac{\tau \cdot R}{I}$$

$\therefore$ small $I \longrightarrow$ increasing a.

$\therefore$ sphere first.

(b). ① $P.E. = Mg \cdot h$

② $K.E. = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$

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hoop:

$$Mgh = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}Mv^2$$

$$\cancel{M}gh = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = \cancel{M}v^2$$

$$v = \sqrt{gh} = \sqrt{10 \cdot 0.52} = 2.28 \text{ m/s}.$$

disk:

$$\cancel{M}gh = \frac{1}{2} \cdot \frac{1}{2}\cancel{M}\cancel{R^2} \cdot \frac{v^2}{\cancel{R^2}} + \frac{1}{2}\cancel{M}v^2$$

$$gh = \frac{1}{4}v^2 + \frac{1}{2}v^2 = \frac{3}{4}v^2$$

$$v = \sqrt{\frac{4}{3}gh} = 2.63 \text{ m/s}.$$

sphere:

$$Mgh = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{5}\cancel{M}\cancel{R^2}\frac{v^2}{R^2} + \frac{1}{2}Mv^2$$

$$gh = \frac{1}{5}v^2 + \frac{1}{2}v^2 = \frac{7}{10}v^2$$

$$v = \sqrt{\frac{10}{7}gh} = 2.73 \text{ m/s}$$

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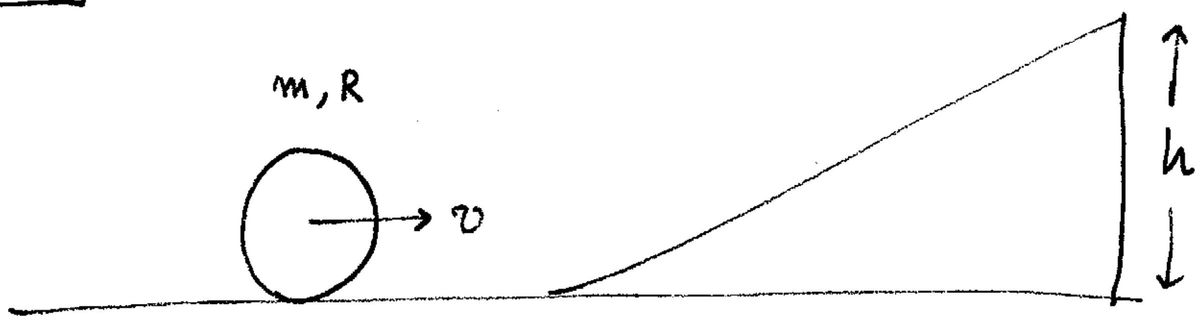
2.



(a) $h = \frac{3}{4}\frac{v^2}{g}$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mg \cdot h$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I \cdot \frac{v^2}{R^2} = mg \cdot \frac{3}{4}\frac{v^2}{g}$$

$$I \cdot \frac{v^2}{R^2} = \frac{3}{2}mv^2 - mv^2 = \frac{1}{2}mv^2$$

$$I = \frac{1}{2}mR^2.$$

(b) $\therefore$ Disk.

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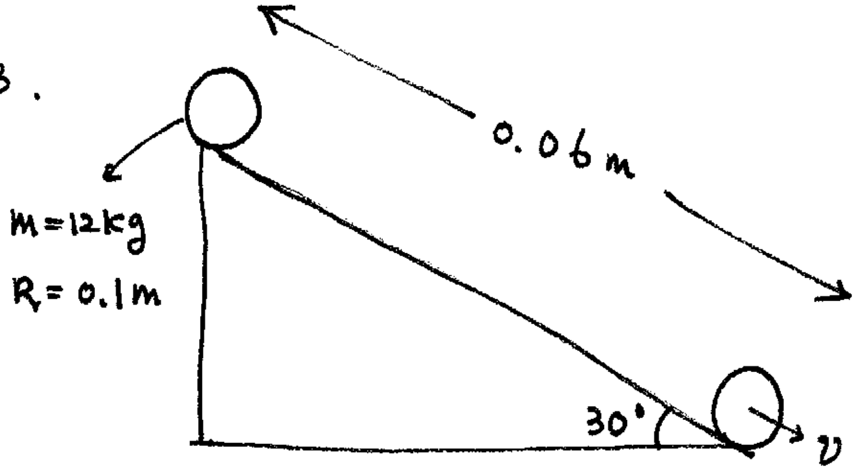
3.



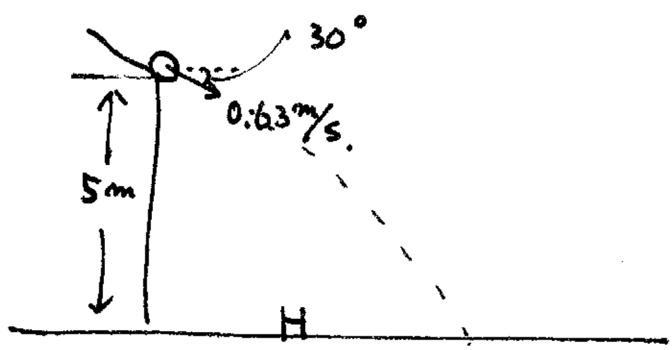
(a) $\cancel{m}gh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{1}{2}m\cancel{R^2}\cdot\frac{v^2}{\cancel{R^2}}$

$= \frac{3}{4}\cancel{m}v^2$

$v = \sqrt{\frac{4}{3}gh} = 0.63 \text{ m/s}.$

$\therefore \omega = \frac{v}{R} = \frac{0.63}{0.1} = 6.3 \text{ rad/s}.$

(b)



$y = v_0 t + \frac{1}{2}at^2$

$5 = 0.63 \cdot \sin 30° + 5 \cdot t^2$

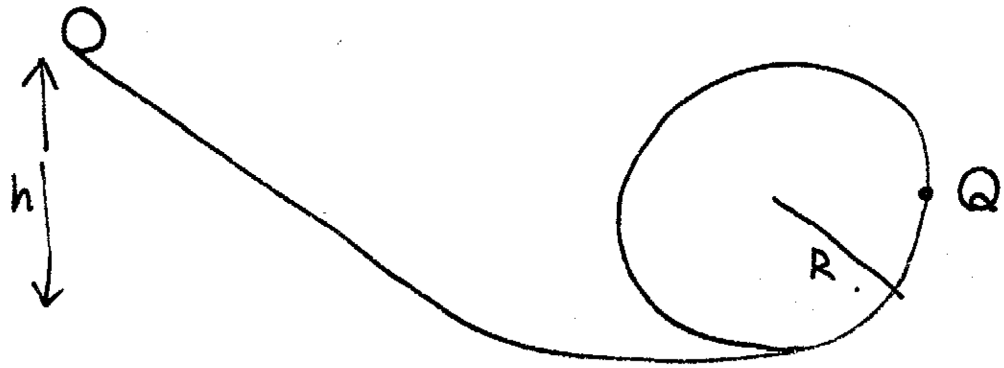
$\therefore t = 0.968 \text{ sec}.$

$H = v_0 t = 0.63 \cdot \cos 30° \cdot (0.968)$

$= 0.53 \text{ m}.$

$I = \frac{2}{5} m r^2$

4.



(a) $mgh = mg(2R) + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

$F = \cancel{m} \frac{v^2}{R} = \cancel{m} g$

$v = \sqrt{Rg}$

$\therefore mgh = 2mgR + \frac{1}{2} \cdot m \cdot Rg + \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{5} m \cdot R \cdot g$

$\cancel{g}h = 2gR + \frac{1}{2} gR + \frac{1}{5} gR = \frac{20+5+2}{10} gR = \frac{27}{10} \cancel{g}R$

$\therefore h = 2.7R$

(b). $\cancel{mg}(6R) = \cancel{mg} R + \frac{1}{2} \cancel{m} v^2 + \frac{1}{\cancel{2}} \frac{\cancel{2}}{5} \cancel{m} v^2$

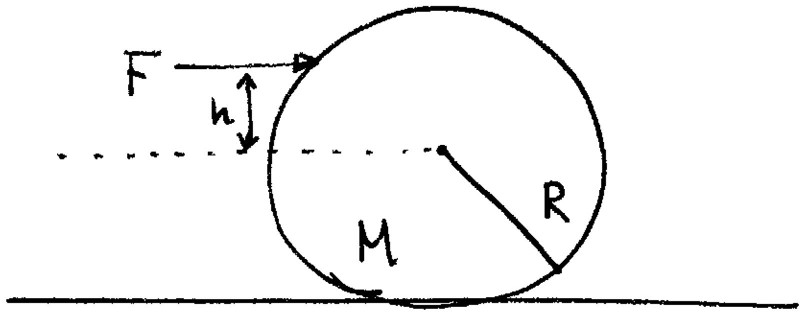
$6gR = gR + \frac{7}{10} v^2$

$5gR = \frac{7}{10} v^2$

$v^2 = \frac{50}{7} gR$

$F = m \frac{v^2}{R} = m \frac{1}{\cancel{R}} \frac{50}{7} g \cancel{R} = \frac{50}{7} mg$

5.



$I = \frac{2}{5} M R^2$.

$F = ma$.

$F = M \cdot a$.

$\tau = I \cdot \alpha$

$F \cdot h = \frac{2}{5} M R^2 \frac{a}{R}$

$\cancel{M} \cancel{a} \cdot h = \frac{2}{5} \cancel{M} \cdot R \frac{\cancel{a}}{1}$

$h = \frac{2}{5} R$