807 AP Physics Schmidt

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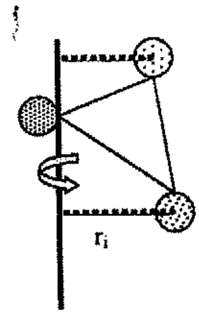
U6H3 **Calculating the Moment of inertia for a discrete mass distribution**

Moment of Inertia I is defined for a rigid object rotating about a fixed axis.

$$I = \sum_i m_i r_i^2$$

r_i is the perpendicular distance of the mass m_i from the axis

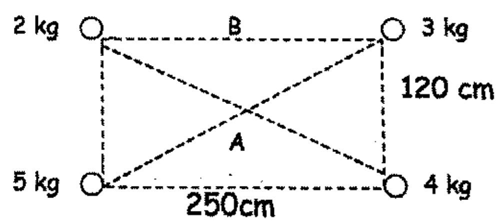


Problems:

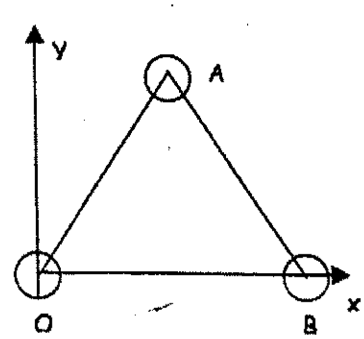
1. A hydrogen chloride molecule consists of a hydrogen atom whose mass m_H is 1.01 u and a chlorine atom whose m_{Cl} is 35.0 u. The centers of the two atoms are a distance d = 127 pm (pico meter=10^{-12}m). What is the rotational inertia of the molecule about an axis perpendicular to the line joining the two atoms and passing through the center of mass of the molecule?

(15800upm^2)

2. Find the moment of inertia of the four masses shown in Fig. relative to an axis perpendicular to the page and extending through points A and through point B (two calculations).
(27 kg m^2; 34kg m^2)



3. Three particles each of mass "m" are situated at the vertices of an equilateral triangle OAB of length "a" as shown in the figure. Calculate moment of inertia (i) about an axis passing through "O" and perpendicular to the plane of triangle (ii) about axis Ox and (iii) about axis Oy.



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1. Calculate the Moment of Inertia of a uniform thin rod of length L and mass M about an axis passing through an end and perpendicular to the rod.

2. Calculate the Moment of Inertia of a uniform thin rod of length L and mass M about an axis passing through its center and perpendicular to the rod.

3. MI of a rectangular plate of dimensions a x b about a line parallel to one of the sides and passing through the center.

4. MI of a circular ring of radius R and mass M about a perpendicular line passing through the center.

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5. MI of a thin circular plate of radius R and mass M about a perpendicular line passing through the center.

6. Moment of inertia of a hollow cylinder of radius R, length L and mass M about its longitudinal axis.

7. Moment of inertia of a uniform solid cylinder of radius R, length L and mass M about its longitudinal axis.

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(EXTRA CREDIT)

8. Moment of inertia of a hollow sphere of radius R and mass M about a diameter.

9. Moment of inertia of a uniform solid sphere of radius R and mass M about a diameter.

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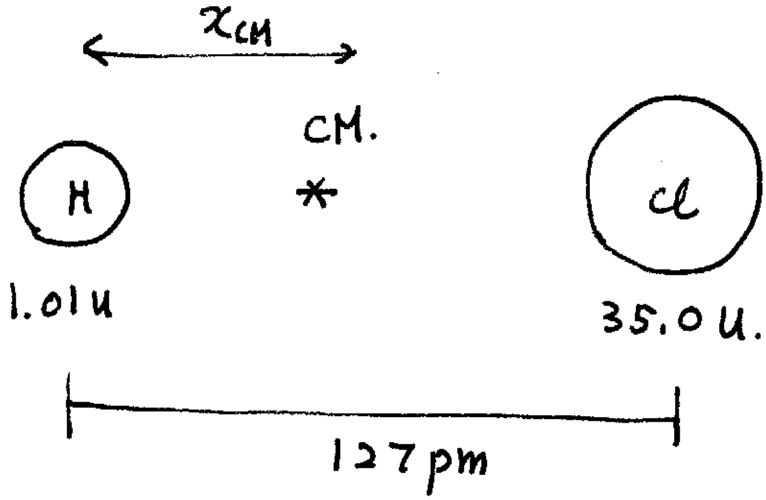
1.



$$(35+1.01)\,x_{CM} = 35 \times 127$$

$$x_{CM} = 123.44 \text{ pm}.$$

$$\Sigma r^2 m = (123.44)^2 \cdot (1.01) + (127-123.44)^2 (35)$$

$$= 15833 \text{ upm}^2$$

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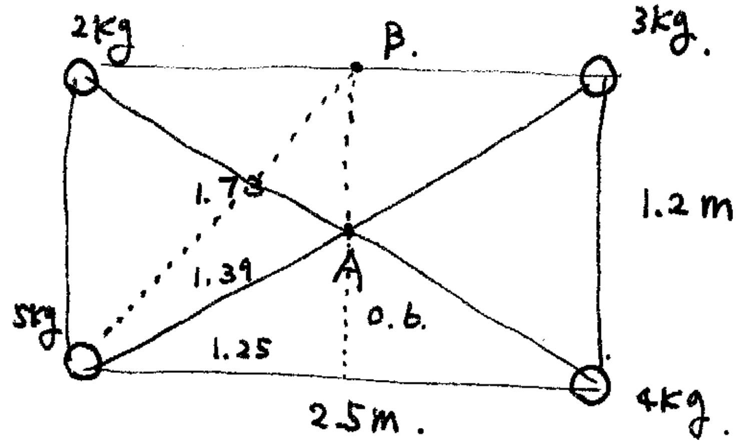
2. A.



$I_A = 5 \times 1.39^2 + 4 \times 1.39^2 + 3 \times 1.39^2 + 2 \times 1.39^2$

$= 27.\ kg\,m^2.$

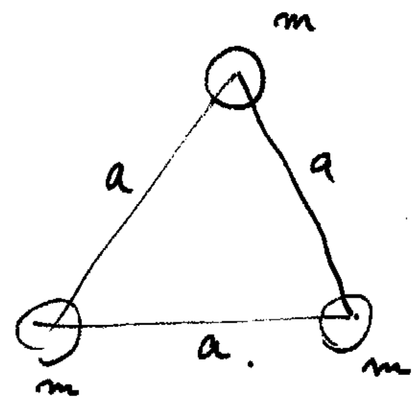
$I_B = 5 \times 1.73^2 + 4 \times 1.73^2 + 3 \times 1.25^2 + 2 \times 1.25^2$

$= 34.75\ kg \cdot m^2$

$\boxed{\text{U 6H3}}$

3. (i).

$$I = a^2 \cdot m + a^2 m$$
$$= 2a^2 m$$

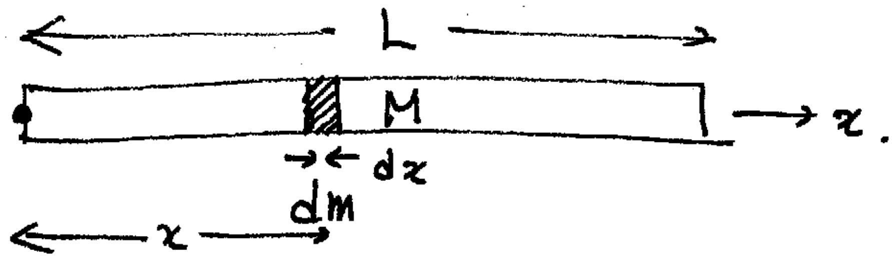


(ii) $I_{ox} = \left(\frac{a}{2}\sqrt{3}\right)^2 \cdot m = \frac{3}{2} a^2 m.$

(iii) $I_{oy} = a^2 m + \left(\frac{a}{2}\right)^2 \cdot m$

$$= \frac{5}{4} a^2 m$$

1.



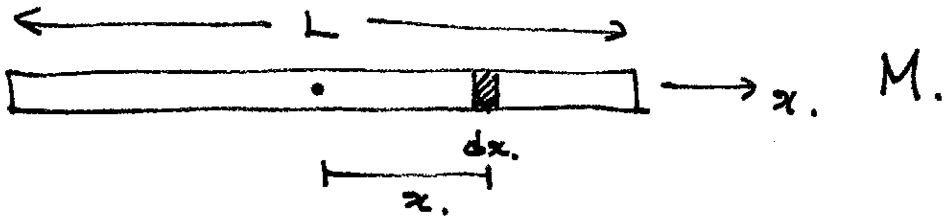
$$dm = \lambda \cdot dx.$$

$$I = \int_0^L x^2 \cdot dm = \int_0^L x^2 \cdot \lambda \, dx = \frac{1}{3} \lambda L^2$$

$$M = \lambda \cdot L$$

$$\therefore \; I = \frac{1}{3} \lambda L L = \frac{1}{3} M L$$

2.

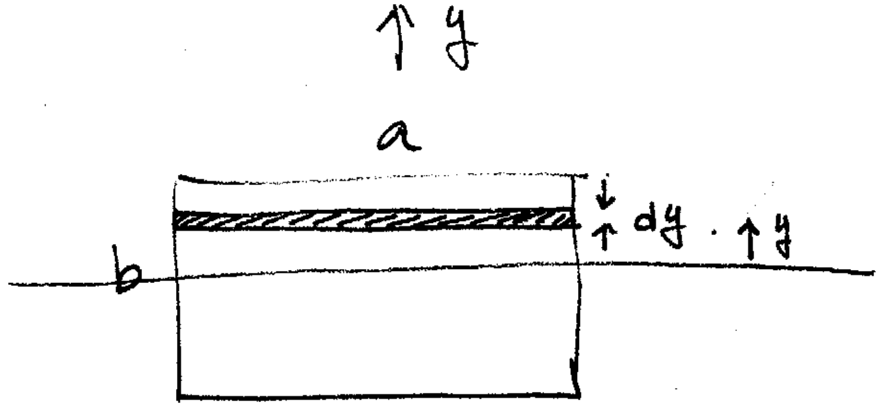


$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \, dm = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \cdot \lambda \cdot dx. = \lambda \ \frac{1}{3} \left[x^3 \right]_{-\frac{L}{2}}^{\frac{L}{2}}$$

$$= \frac{1}{12} \lambda L^3 = \frac{1}{12} ML^2$$

$*. \ M = \lambda L$

3.



$$dm = \rho \cdot a \cdot dy$$

$$M_{Total} = \rho \cdot ab.$$

$$I = \int_{-\frac{b}{2}}^{\frac{b}{2}} y^2 \cdot dm = \int_{-\frac{b}{2}}^{\frac{b}{2}} y^2 \cdot \rho \cdot a \cdot dy$$

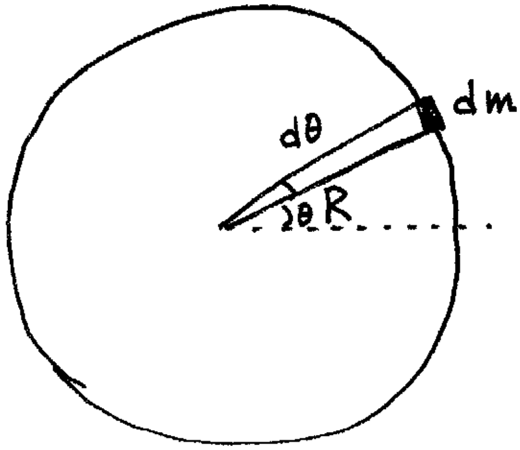
$$= \rho \cdot a \cdot \int_{-\frac{b}{2}}^{\frac{b}{2}} y^2 \, dy$$

$$= \rho \cdot a \cdot \frac{b^3}{12} = \rho \cdot a \cdot b \cdot \frac{b^2}{12}$$

$$= \frac{1}{12} M b^2$$

4.

$$M = \lambda 2\pi R$$

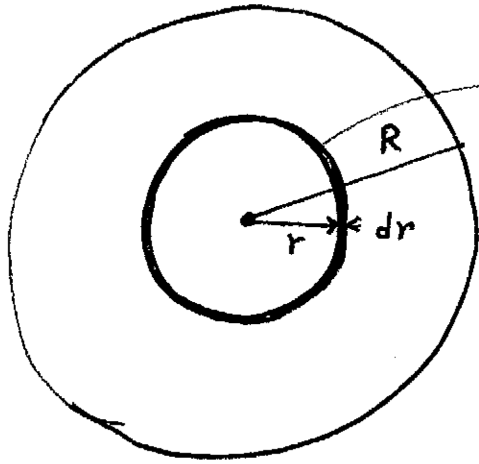


$$I = \int_0^{2\pi} R^2 \cdot dm = R^2 \int_0^{2\pi} \lambda \, ds = R^2 \lambda \int_0^{2\pi} R \cdot d\theta$$

$$= R^3 \lambda \cdot 2\pi.$$

$$= MR^2$$

5.



$$dI = dm\, r^2$$
$$= \rho \cdot 2\pi r \cdot r^2\, dr$$
$$= 2\pi\rho \cdot r^3\, dr$$

$$I = \int_0^R r^2\, dm = \int_0^R 2\pi\rho \cdot r^3\, dr$$

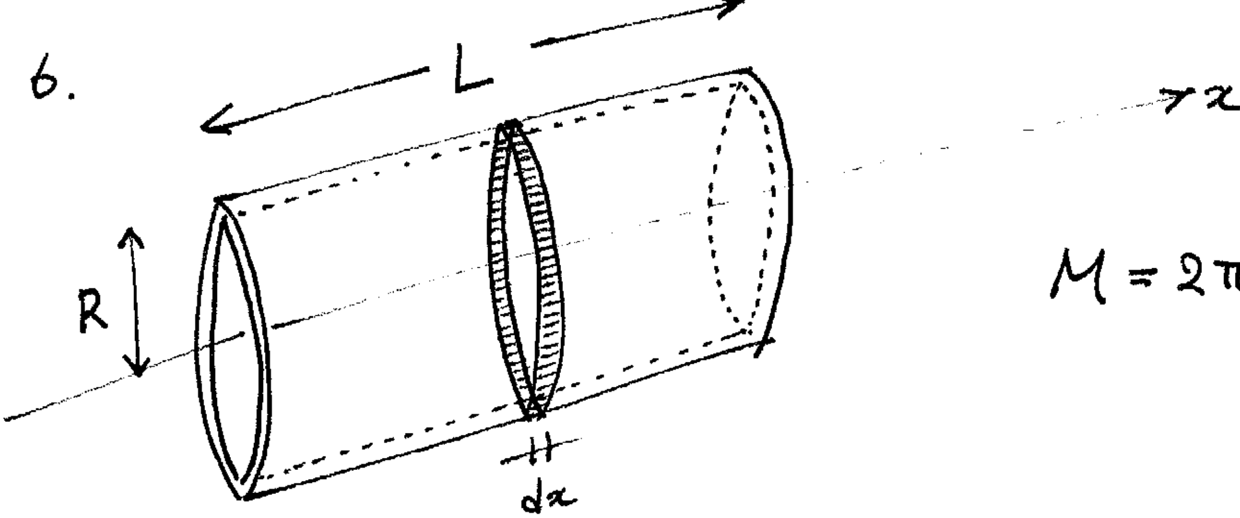
$$= 2\pi\rho\, \frac{1}{4} R^4 = \frac{\pi\rho}{2} R^4$$

$$M = \rho\pi R^2$$

$$= \frac{1}{2}\rho\pi R^2 \cdot R^2$$

$$= \frac{1}{2} M R^2$$

6.



$$M = 2\pi R L \rho$$

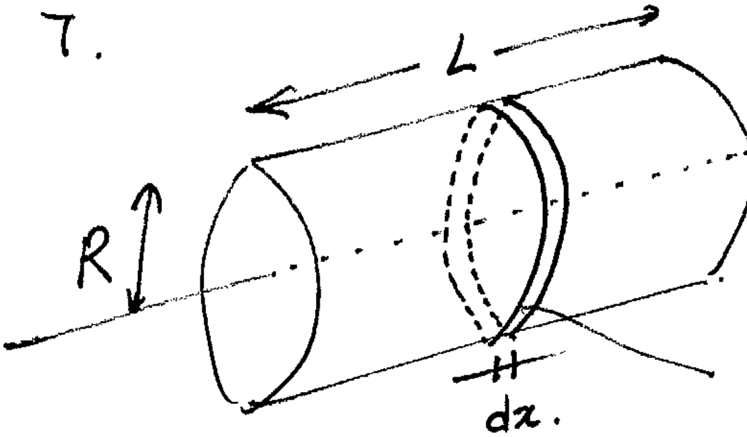
$$I = \int_0^L dI = \int_0^L dm\, R^2 = R^2 \int_0^L \rho\, 2\pi R \cdot dx$$

$$= 2\pi R^3 \rho\, L.$$

$$= 2\pi R L \rho \cdot R^2$$

$$= MR^2$$

7.



$$dI = \frac{1}{2} dm\, R^2$$

$$= \frac{1}{2} \rho \cdot \pi R^2 \cdot R^2\, dx.$$

$$I = \int_0^L \frac{1}{2} \rho \pi R^4\, dx = \frac{1}{2} \rho \pi R^4 L$$

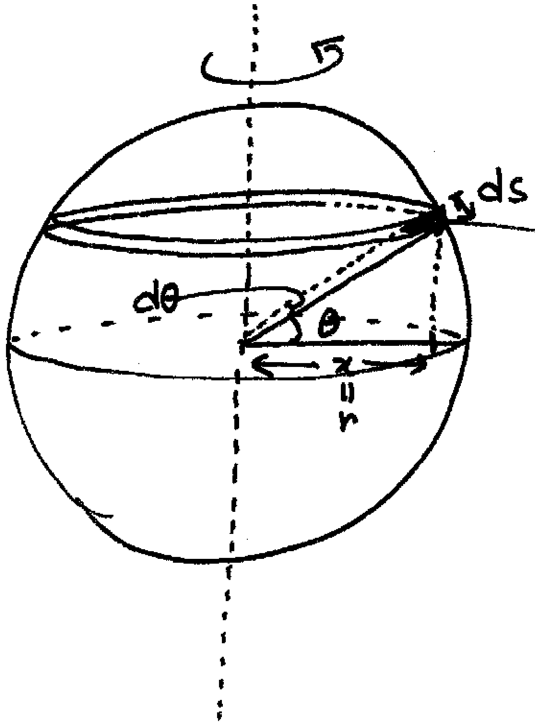
$$* \; M = \rho \pi R^2 \cdot L$$

$$= \frac{1}{2} \rho \pi R^2 \cdot L \cdot R^2$$

$$= \frac{1}{2} M R^2.$$

W6H3

8.



$$dm = \rho \cdot dV$$

$$= \rho \cdot ds \cdot 2\pi \cdot x \cdot t$$

$$= \rho \cdot (R \cdot \cos\theta)\, R \cdot d\theta \cdot 2\pi \cdot t$$

$$= \rho \cdot R^2 \cdot 2\pi t \cdot \cos\theta \cdot d\theta$$

$$\boxed{* \; M = \rho \cdot 4\pi R^2 \cdot t}$$

$$I = \int dm \cdot r^2$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \rho \cdot R^2 \cdot 2\pi t \cdot \cos\theta \cdot (R\cos\theta)^2 \, d\theta$$

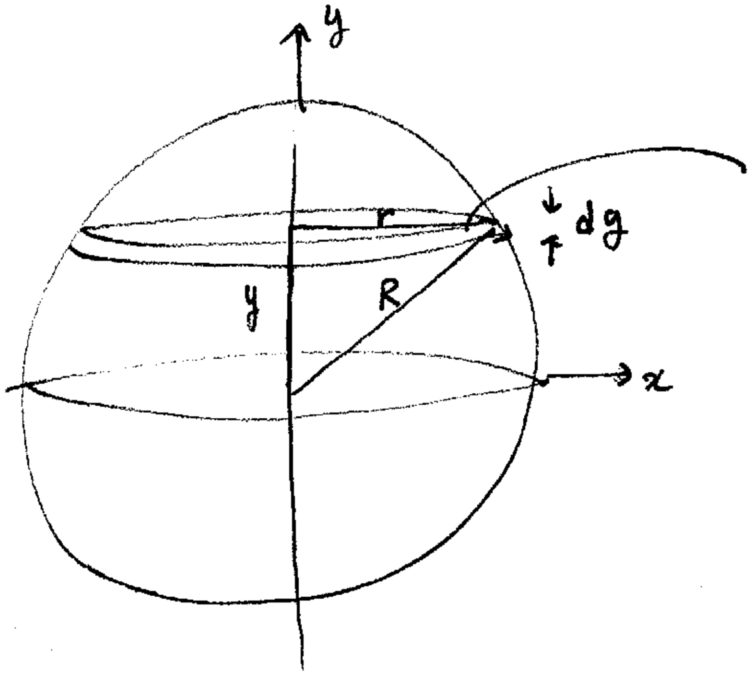
$$= R^4 \cdot \rho \cdot 2\pi \cdot t \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos\theta)^3 \, d\theta$$

$$= \rho \cdot 4\pi R^2 \cdot t \cdot \frac{R^2}{2} \cdot \frac{4}{3}$$

$$= M \cdot \frac{2}{3} R^2 = \frac{2}{3} MR^2$$

2x

9.



$$dI = \frac{1}{2} dm\, r^2$$

$$= \frac{1}{2} \rho \cdot \pi r^2 \cdot r^2 \cdot dy.$$

$\divideontimes$. $r^2 + y^2 = R^2.$

$r^2 = R^2 - y^2$

$$= \frac{1}{2} \rho \pi (R^2 - y^2)^2 dy.$$

$$I = \int_{-R}^{R} \frac{1}{2} \rho \pi (R^2 - y^2)^2 \cdot dy = \frac{1}{2} \rho \pi \int_{-R}^{R} (R^2 - y^2)^2 dy$$

$$= \frac{1}{2} \rho \pi \quad \frac{16}{15} R^5$$

$\divideontimes$ $M = \rho \frac{4}{3} \pi R^3$

$$= \frac{1}{2} \rho \pi \frac{4}{3} R^3 \quad \frac{4}{5} R^2$$

$$= \frac{2}{5} M R^2$$